Degree of interdependence of cryptocurrency quotes assessment using Dynamic Time Warping

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Presentation outline

Cryptocurrency market

Goal and motivation

Research methods

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Empirical analysis

Interpretation, conclusions. Directions for further research

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Cryptocurrency

Cryptocurrency exchange:

- operate continuously
- require dedicated, fast algorithms that allow for effective on-line transactions

Cryptocurrency market:

- Cryptocurrency - a distributed currency system based on cryptography
- Bitcoin (BTC) - the most popular cryptocurrency (2009)
- Altcoins - over 19,000 other cryptocurrencies, e.g. Litecoin (LTC, 2011), Ethereum (ETH, 2015), USD Coin (USDC, 2016)
- around 70 has a market capitalisation exceeding 1 billion USD
- dozens cryptocurrency exchanges

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Top 10 cryptocurrencies by market capitalization

#	Name		Price	1h %	24h %	7d %	Market Cap ⓘ	Volume(24h) ⓘ	Circulating Supply ⓘ	Last 7 Days
1	🟠 Bitcoin BTC	Buy	$110,777.95	▼ 0.30%	▲ 3.24%	▲ 8.76%	$2,200,966,784,999	$91,374,652,796 822.34K BTC	19.86M BTC	📈
2	◆ Ethereum ETH	Buy	$2,658.56	▲ 0.82%	▲ 4.62%	▲ 6.16%	$320,960,288,182	$37,777,216,564 14.20M ETH	120.72M ETH	📈
3	🟢 Tether USDT	Buy	$0.9999	▲ 0.02%	▼ 0.04%	▼ 0.01%	$152,394,296,892	$146,460,872,739 146.49B USDT	152.39B USDT	📉
4	✖ XRP XRP	Buy	$2.42	▲ 0.47%	▲ 2.62%	▲ 0.55%	$142,566,882,895	$4,492,435,942 1.85B XRP	58.68B XRP	📉
5	🟡 BNB BNB	Buy	$686.01	▲ 0.00%	▲ 4.68%	▲ 5.99%	$96,651,211,331	$2,669,684,496 3.89M BNB	140.88M BNB	📈
6	🟣 Solana SOL	Buy	$178.72	▲ 0.48%	▲ 5.42%	▲ 6.61%	$92,966,720,125	$6,599,447,750 36.96M SOL	520.16M SOL	📈
7	🔵 USDC USDC	Buy	$0.9997	▲ 0.02%	▼ 0.01%	▼ 0.01%	$61,054,193,014	$18,756,037,052 18.76B USDC	61.11B USDC	📉
8	🟡 Dogecoin DOGE	Buy	$0.2416	▲ 0.33%	▲ 6.22%	▲ 10.67%	$36,100,151,554	$3,444,900,689 14.27B DOGE	149.36B DOGE	📈
9	⬡ Cardano ADA	Buy	$0.8049	▲ 0.78%	▲ 6.56%	▲ 7.01%	$28,433,101,340	$1,353,328,391 1.68B ADA	35.32B ADA	📈
10	🔴 TRON TRX	Buy	$0.2779	▲ 1.10%	▲ 2.56%	▲ 2.86%	$26,375,675,657	$1,248,752,462 4.50B TRX	94.88B TRX	📈

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Source: https://coinmarketcap.com/

www.sgh.waw.pl

Bitcoin related statistics

- over 300 million crypto users worldwide

- over 18,000 companies worldwide accept Bitcoin as a payment method

- 55% of people look at crypto as a long-term investment strategy

- Bitcoin is posted about on social media every 2 seconds

- Bitcoin mining uses the same energy annually that could power 10 million homes

- 67% of millennials view Bitcoin as a safer place to store wealth compared with gold

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Historical BTC/USD quotes in 2013-2025

100.00K

85.04K

75.00K

50.00K

25.00K

0.05816

USD

2011 2012 2013 2014 2015 2016 2017 2018 2019 2020 2021 2022 2023 2024 2025

4   2   2   2   2   12   7   74 2

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Source: https://coinmarketcap.com/

www.sgh.waw.pl

Goal and motivation

Motivation:

- **Similarity of behaviour between the cryptocurrencies and assets on traditional financial exchanges.**

Goal:

- **Validation of the Bitcoin leading role over other cryptocurrencies and assets on traditional financial exchanges.**

- **Verifying the possibility of finding a lag pattern reflecting the bear and bull situation on the cryptocurrency market near the Bitcoin quote trend reversal.**

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Research methods

Statistics of co-occurrence of changes

- Taking into account lags.
- Different variants of the magnitude of changes.

Dynamic Time Warping

- Time series similarity measure.
- A more appropriate approach than commonly used measures such as Euclidean distance, correlation, cosine similarity or Jaccard index.

Time series similarity measures
(according to ChatGPT)

- **Euclidean Distance**: It measures the straight-line distance between two points in a multi-dimensional space and is a simple measure of similarity.

- **Correlation**: It measures the relationship between two series and is used to determine the strength and direction of the relationship.

- **Cosine Similarity**: It is a measure of similarity between two non-zero vectors of an inner product space. It measures the cosine of the angle between two vectors.

- **Jaccard Similarity**: It is a measure of similarity between two sets and is defined as the size of the intersection divided by the size of the union of the sets. It is commonly used for comparing sequences with categorical variables.

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Dynamic Time Warping (DTW)

HISTORY – introduced independently in (Vintsyuk, 1968) and (Sakoe & Chiba, 1978) in the area of speech recognition

IDEA – measure of the differences between points in a time series and finding the optimal shift of the time series to minimize the total distance

CHARACTERISTICS – a powerful tool for time series analysis due to, among other things, the possibility of applying it to data of varying length and variability

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Dynamic Time Warping – applications

Speech recognition:

align speech recordings to a reference template to improve speech recognition systems' accuracy

Bioinformatics:

align biological sequences, such as DNA and proteins, to differentiate or compare them

Music analysis:

align different performances of a piece of music and to identify similarities and differences between them.

Monitoring:

align time-series data from various sensors, to detect anomalies in the data and monitor the performance of systems

Robotics:

align sensor data from robots, such as accelerometer and gyroscope readings, to recognise and track gestures

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Dynamic Time Warping – example (1)

Walking sequence recorded using a motion capture system
Source: https://en.wikipedia.org/wiki/Dynamic_time_warping
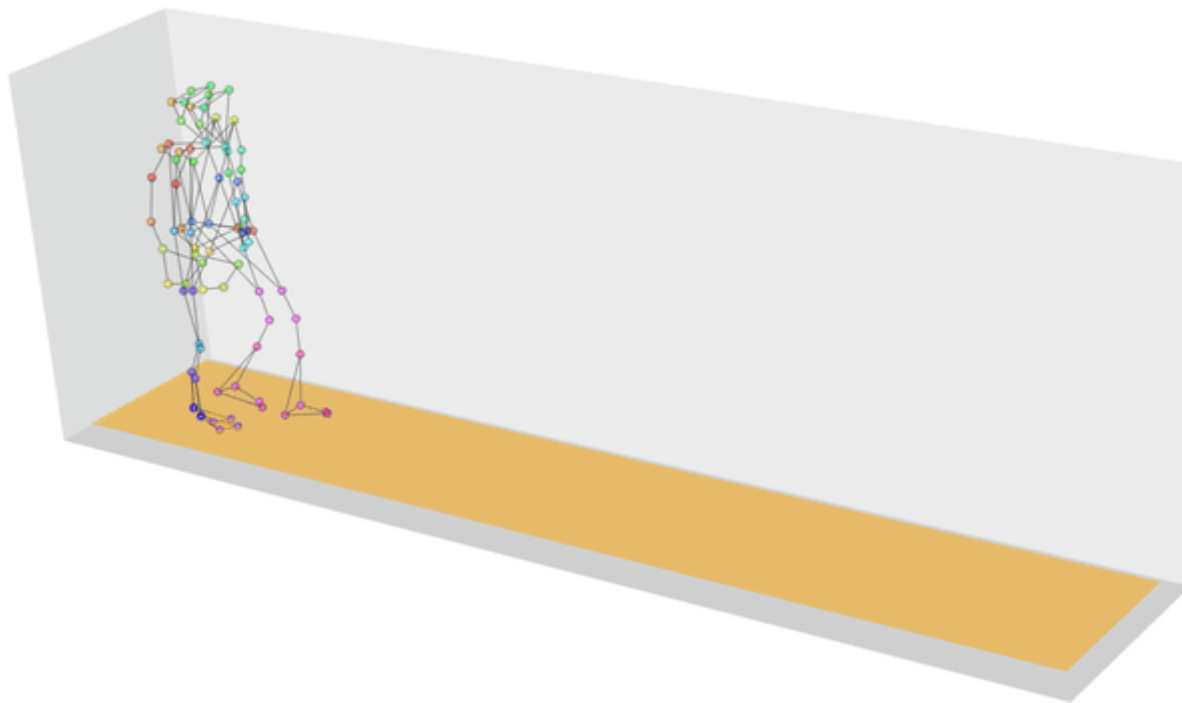
Dynamic Time Warping – example (2)
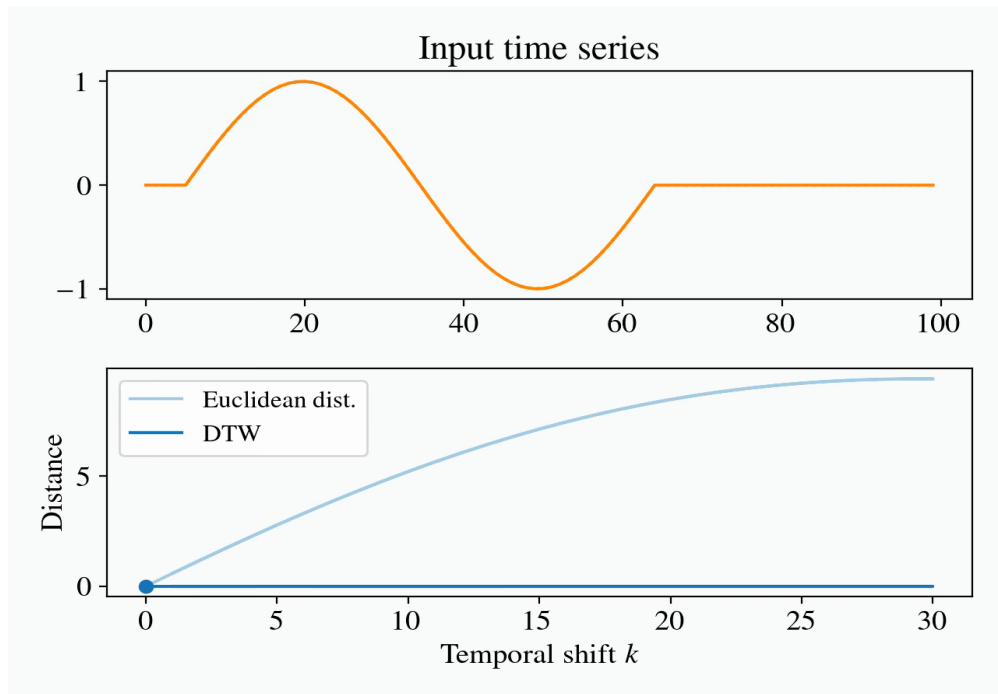
Euclidean distance vs DTW:

DTW is invariant to time shifts between time series

Source: https://rtavenar.github.io/blog/dtw.html

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Dynamic Time Warping – example (3)
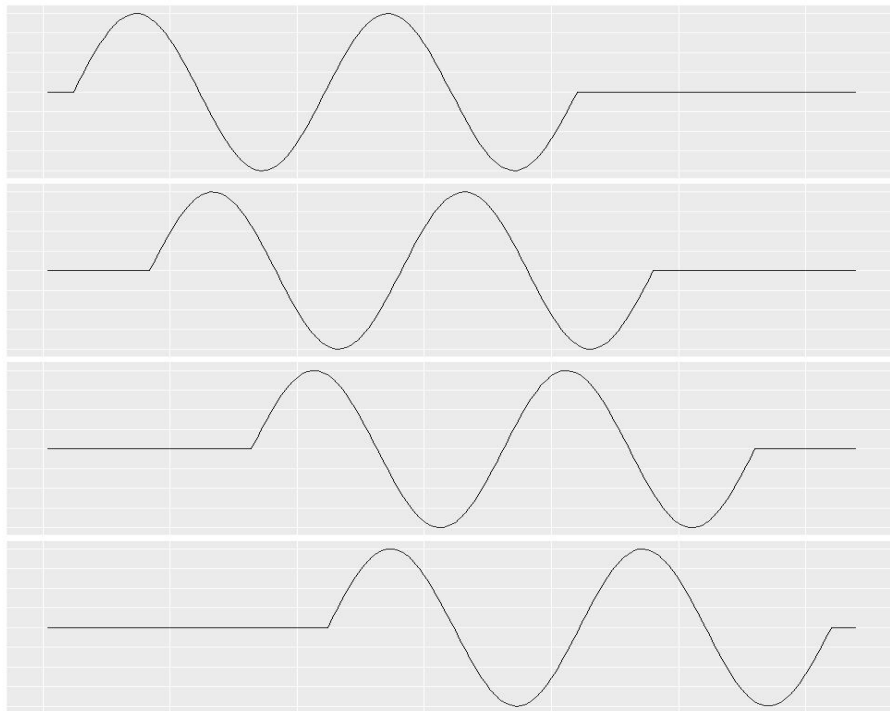
Time series shifted in time relative to each other:

DTW measure = 0

Data characteristics

Website www.investing.com

Daily quotes from 01.01.2021 to 21.05.2025

Skipping the initial period of interest in cryptocurrencies

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List of assets

Bitcoin (BTC)

Dow Jones U.S. Oil & Gas Index (OIL)

Gold Spot US Dollar Price (XAU)

Ethereum (ETH)

Tether (USDT)

XRP (XRP)

BNB (BNB)

Solana (SOL)

USDC (USDC)

Dogecoin (DOGE)

Cardano (ADA)

Tron (TRX)

Research procedure

Relative change (day to the previous quotation day)

Clustering depending on the direction and magnitude of relative changes

Quantification of the degree of similarity between pairs of assets using statistics of co-occurrence and DTW measure

Bitcoin behaviour – turning points identification
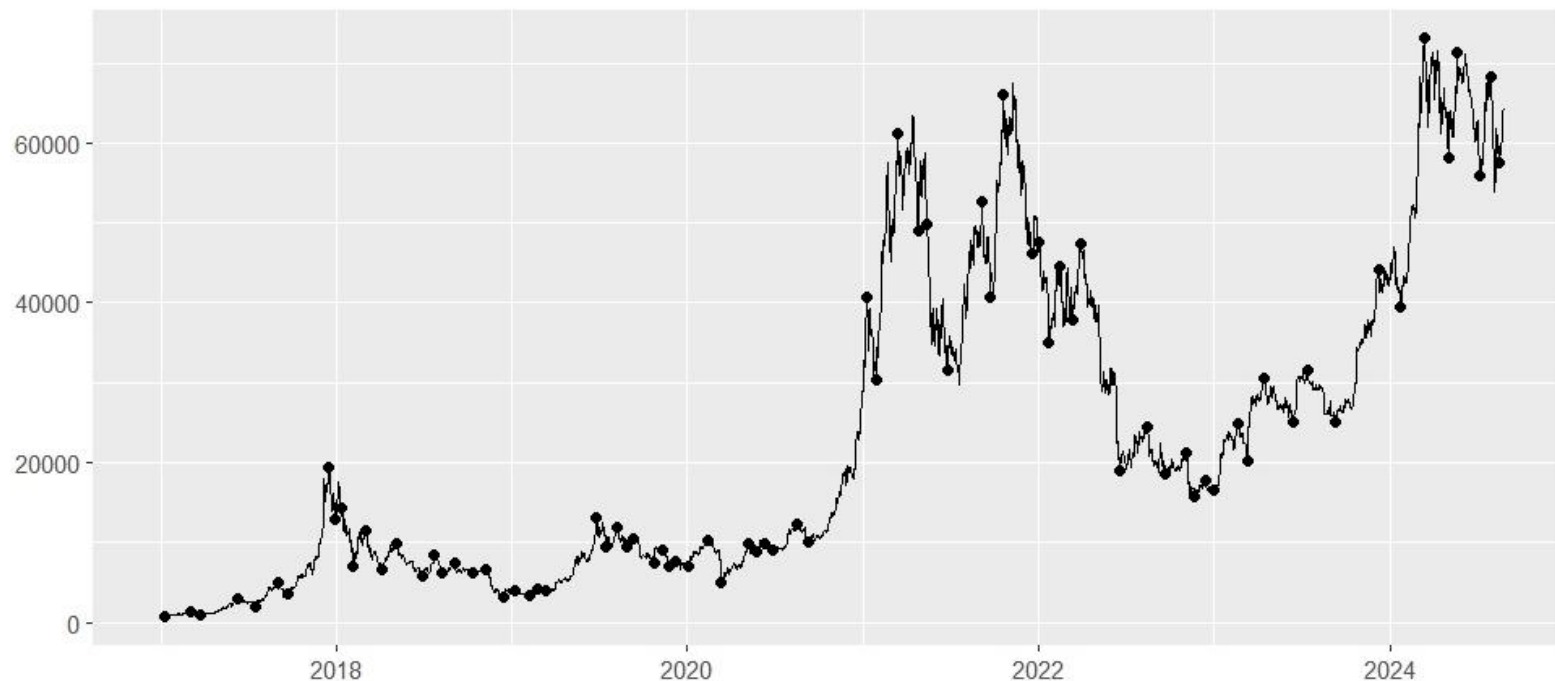
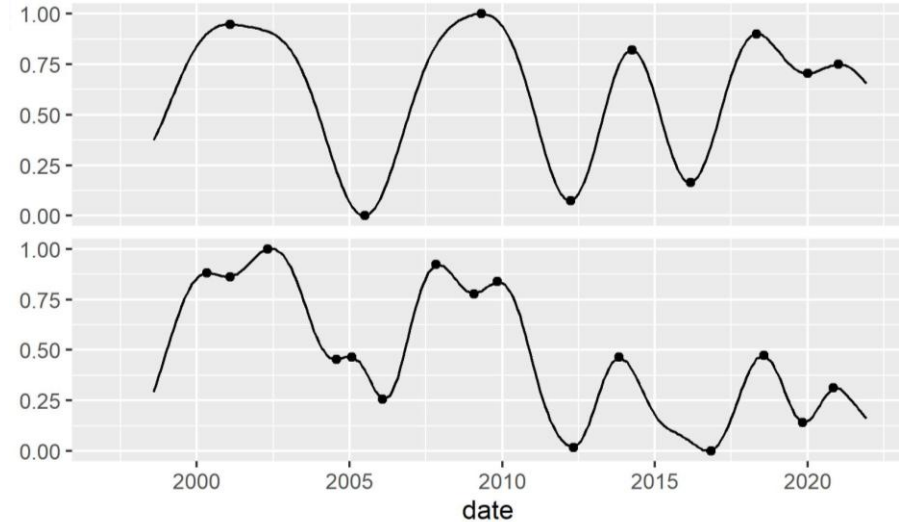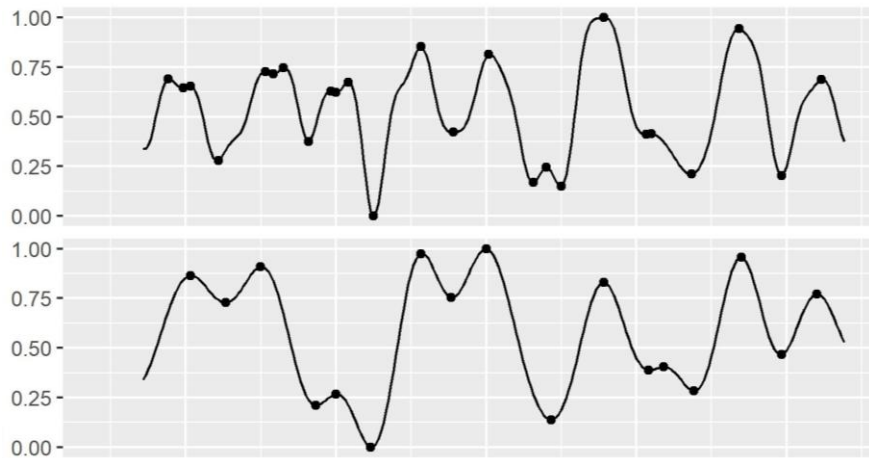# Results differ on chosen parameters and assumptions

Values since January 2025
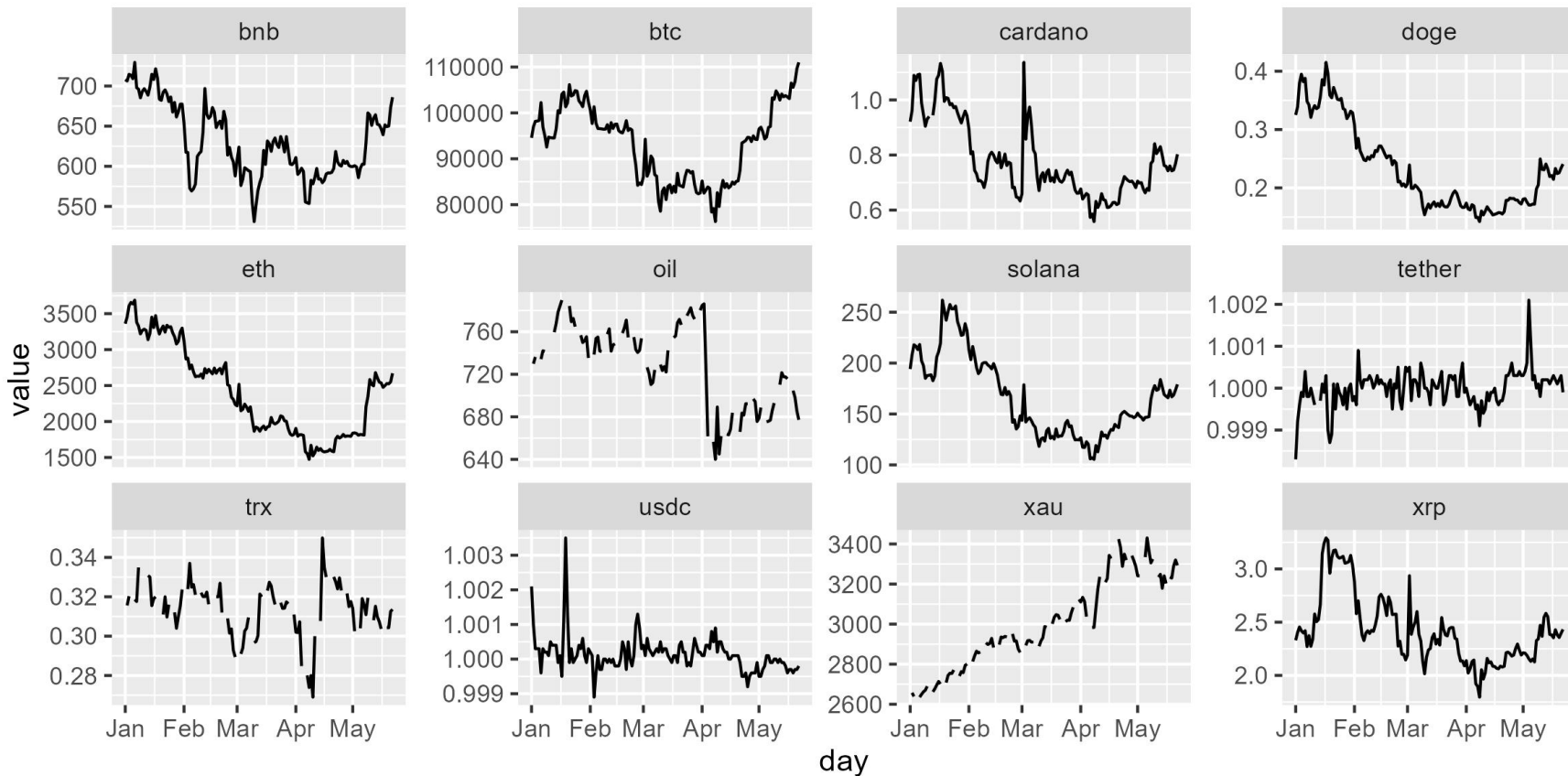
Relative changes since April 2025 (after normalization)
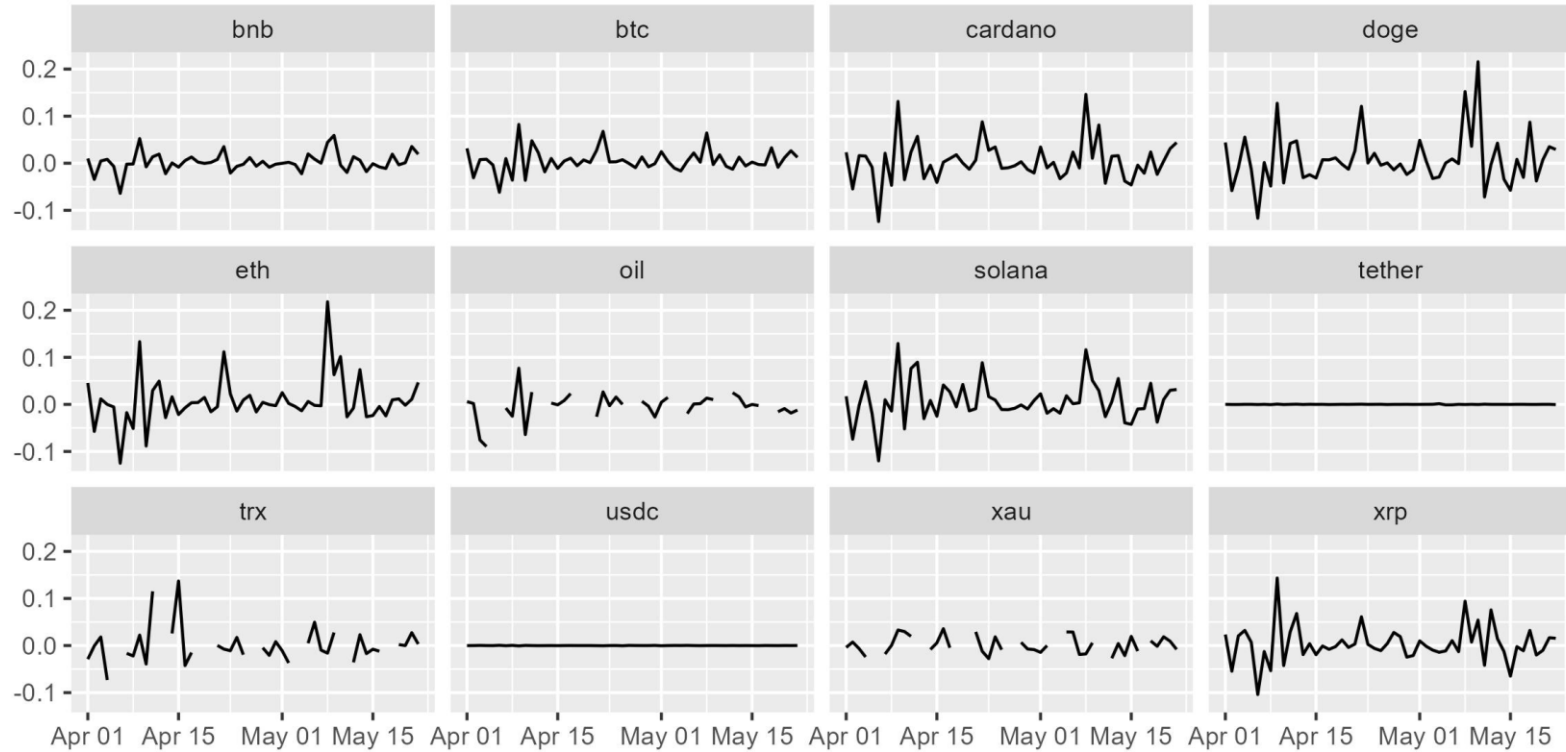
Selected variants

asset	mean	min	Q1	median	Q3	max
bnb	0.0028	-0.3407	-0.0146	0.0010	0.0173	0.6999
btc	0.0013	-0.1563	-0.0133	0.0000	0.0156	0.1941
cardano	0.0023	-0.2662	-0.0234	-0.0002	0.0223	0.7242
doge	0.0060	-0.3949	-0.0264	-0.0004	0.0245	3.8737
eth	0.0017	-0.2789	-0.0180	0.0009	0.0209	0.2596
oil	0.0008	-0.0895	-0.0088	0.0014	0.0107	0.0770
solana	0.0048	-0.4235	-0.0293	0.0004	0.0334	0.6486
tether	0.0000	-0.0034	-0.0002	0.0000	0.0002	0.0035
trx	-0.0001	-0.1456	-0.0195	-0.0013	0.0160	0.2652
usdc	0.0000	-0.0341	-0.0001	0.0000	0.0002	0.0252
xau	0.0005	-0.0349	-0.0045	0.0006	0.0057	0.0359
Xrp	0.0029	-0.3325	-0.0197	0.0008	0.0209	0.7300

two categories: asset value rise and fall

three categories: fall below Q1, moderate changes Q1-Q3, rise over Q3

three categories: fall below 10%, moderate changes in the range 10-90%, rise over 90%

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Accuracy – two categories: asset value rise and fall

Asset	No lag	1 day lag	2 days lag	3 days lag	4 days lag	5 days lag	DTW
ADA	78.76	47.62	49.78	51.81	49.97	47.31	29.00
BNB	77.40	46.78	49.12	51.16	50.06	47.28	12.00
DOGE	78.03	46.28	49.44	51.47	50.50	47.84	1.00
ETH	81.90	46.53	49.88	52.22	48.87	48.22	0.00
OIL	55.90	51.27	50.36	51.54	48.05	50.82	51.00
SOL	77.01	47.31	50.22	50.81	50.85	48.37	29.00
TRX	56.65	48.87	49.32	51.31	50.91	48.41	8.00
USDC	40.79	51.56	53.97	49.25	51.66	50.94	2.00
USDT	57.59	46.65	48.42	48.99	48.73	48.42	11.00
XAU	54.53	48.67	50.71	51.42	51.6	51.78	10.00
XRP	77.29	45.92	50.98	50.85	49.34	48.13	1.00

Accuracy – three categories: <Q1, Q1-Q3, >Q3

Asset	No lag	1 day lag	2 days lag	3 days lag	4 days lag	5 days lag	DTW
ADA	66.08	38.62	40.15	38.36	39.82	39.72	124.00
BNB	67.73	41.04	40.19	40.34	40.80	37.76	118.50
DOGE	65.98	39.16	41,00	38.84	39.61	38.57	113.50
ETH	73.22	40.91	39.94	38.71	40.55	39.64	115.50
OIL	41.20	39.20	37.75	39.84	39.69	39.36	141.00
SOL	64.52	39.25	38.59	37.80	39.95	38.66	122.00
TRX	41.18	35.20	39.91	40.18	39.86	37.17	137.00
USDC	35.73	41.62	39.59	40.05	39.57	39.35	148.00
USDT	45.44	39.43	38.16	37.59	37.53	39.30	140.50
XAU	37.30	34.01	38.45	39.43	37.83	39.34	145.50
XRP	64.90	41.24	40.23	40.73	38.77	39.47	129.00

Accuracy – three categories: <10%, 10-90%, >90%

Asset	No lag	1 day lag	2 days lag	3 days lag	4 days lag	5 days lag	DTW
ADA	81.45	67.25	67.17	67.15	68.13	67.29	52.00
BNB	80.02	68.39	67.12	67.48	67.83	66.50	61.50
DOGE	79.90	67.33	67.44	66.54	66.83	66.19	64.50
ETH	85.14	68.71	68.19	68.17	66.71	67.50	49.00
OIL	66.24	66.52	65.88	68.06	67.57	68.36	61.50
SOL	79.70	68.12	68.17	67.71	67.94	67.61	58.00
TRX	66.97	65.97	67.69	66.61	67.75	66.09	66.50
USDC	68.52	69.56	68.23	68.52	68.25	66.67	74.50
USDT	69.37	67.41	67.28	67.53	67.22	67.66	74.00
XAU	64.83	64.83	67.32	67.76	69.27	67.67	62.00
XRP	80.08	68.75	67.99	67.17	68.25	67.17	60.00

Results – rankings

Asset	Variant 1		Variant 2		Variant 3	
	Accuracy ranking	DTW ranking	Accuracy ranking	DTW ranking	Accuracy ranking	DTW ranking
ADA	2	10	3	5	2	2
BNB	4	8	2	3	4	5
DOGE	3	3	4	1	5	8
ETH	1	1	1	2	1	1
OIL	9	11	8	9	10	6
SOL	6	9	6	4	6	3
TRX	8	5	9	7	9	9
USDC	11	4	11	11	8	11
USDT	7	7	7	8	7	10
XAU	10	6	10	10	11	7
XRP	5	2	5	6	3	4

Results – dendrograms
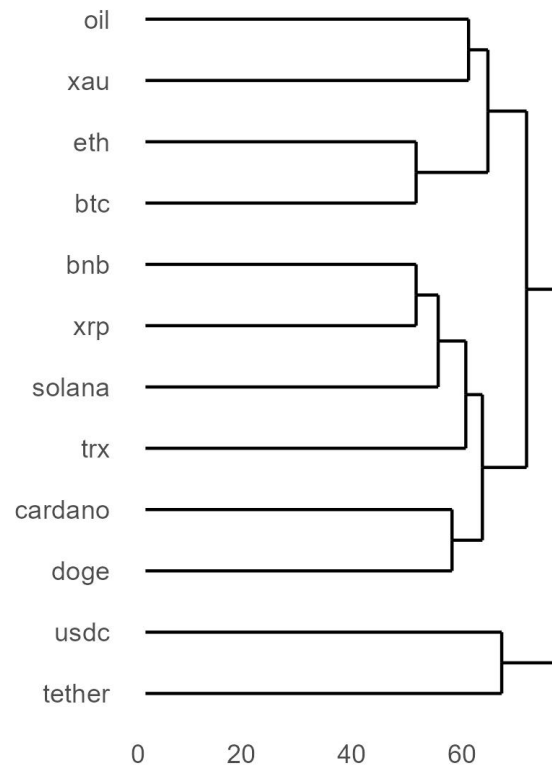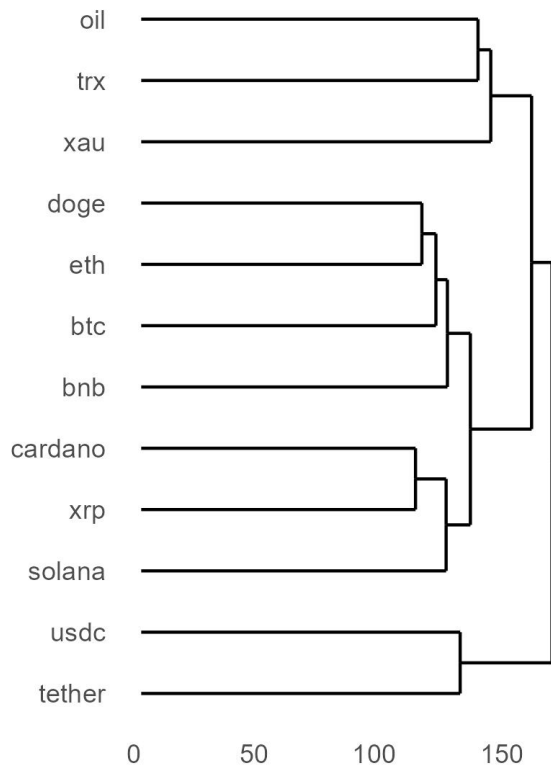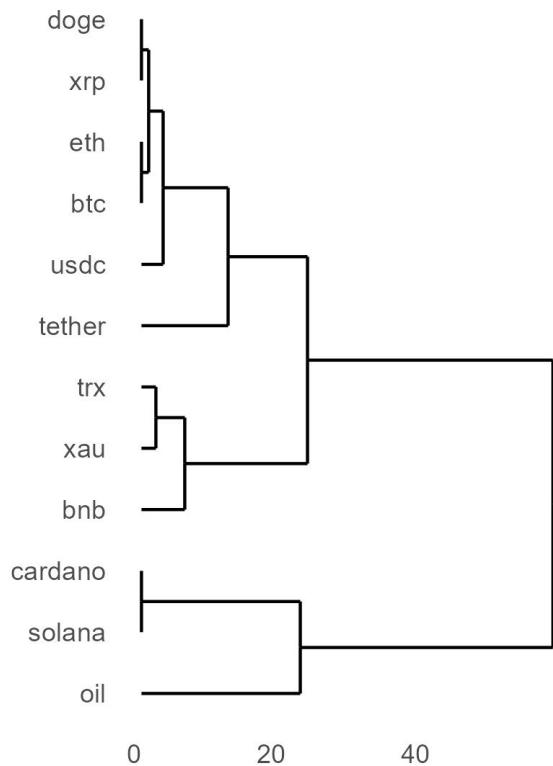
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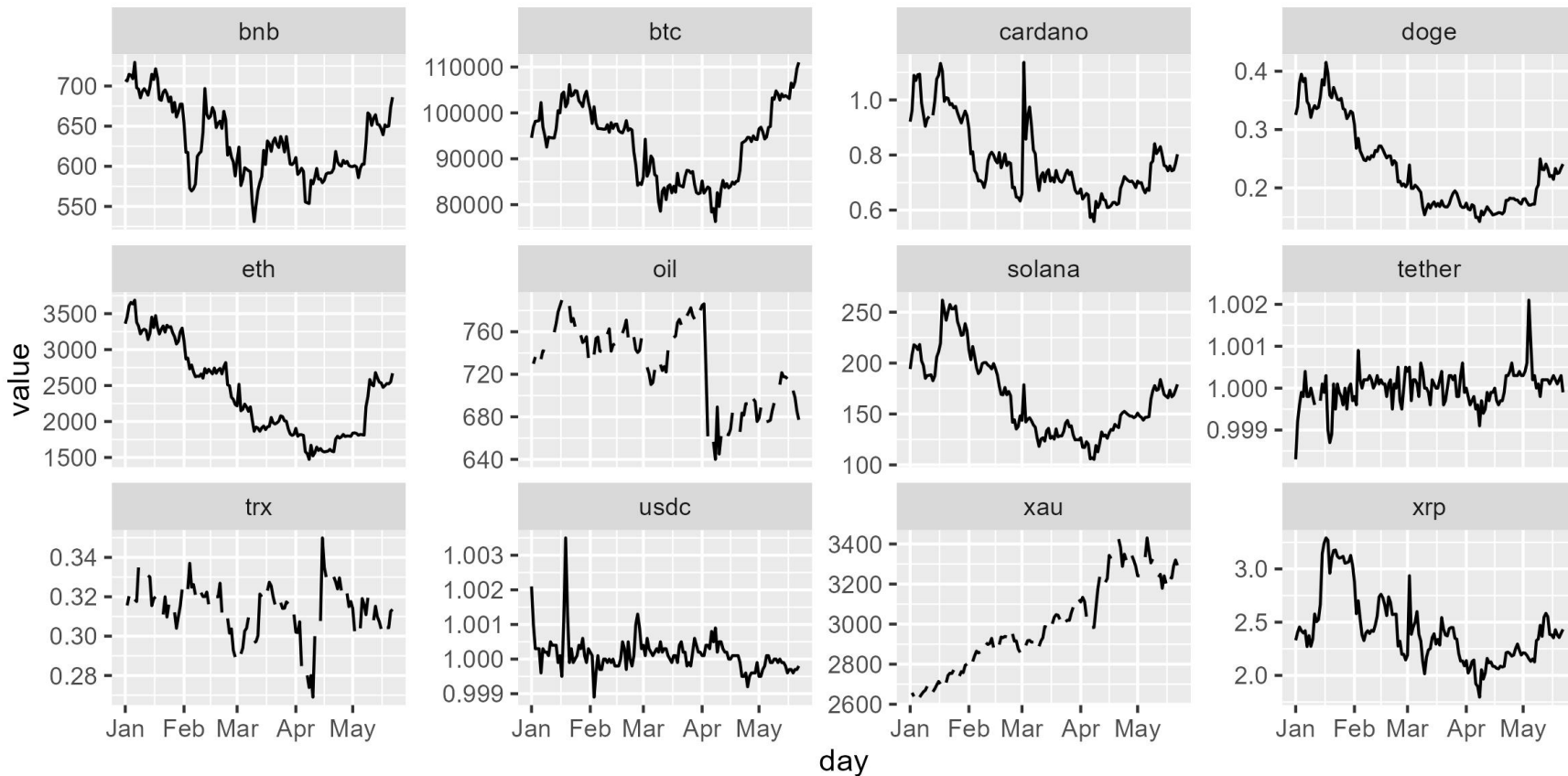
Findings and interpretation

Ethereum (ETH) is closely following Bitcoin price changes

Gold and Dow Jones U.S. Oil & Gas Index have different behaviour patterns

USD Coin (USDC) is a stablecoin that is backed by U.S. dollars and U.S. Treasury instruments – is loosely linked to bitcoin quotes

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Conclusions

Variant with two categories is characterized by too much randomness, and high accuracy is rather related to a high probability of co-occurrence and common trends in the economy.

Variant 2, related to the interquartile range, covers price changes that are not yet significant enough to affect the prices of other assets.

Variant 3 covers only significant rate changes and is the most reliable.

The DTW measure automatically considers time shifts and allows for capturing similarity throughout the considered period.

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Summary

DTW should perform better over shorter periods, preferably between turning points.

Comparison with assets on traditional financial exchanges is difficult due to non-trading days. Excluding periods that include those days should improve the quality of the match.

Due to the high volatility in cryptocurrency markets, daily quotes are sometimes insufficient to identify the existence or magnitude of lags.

Choosing a magnitude of change that will be large enough to impact other assets is essential.

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Thank you for your attention!